$$\mathbb{P}(A^{\complement}) = 1 - \mathbb{P}(A) \qquad \mathbb{P}(B \setminus A) = \mathbb{P}(B) - \mathbb{P}(A \cap B) \qquad \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

$$\mathbb{P}(B|A)\,\mathbb{P}(A) = \mathbb{P}(A \cap B) \qquad \mathbb{P}(B) = \sum_{i=1}^{n} \mathbb{P}(A_i)\,\mathbb{P}(B|A_i) \qquad \mathbb{P}(A_i|B) = \frac{\mathbb{P}(B|A_i)\,\mathbb{P}(A_i)}{\sum_{k=1}^{n} \mathbb{P}(B|A_k)\,\mathbb{P}(A_k)}$$

| Förd. | pmf(k) | cdf $= \mathbb{P}(X \leq k)$ | $\mathbb{E}[X] = \mu$ | $\mathrm{Var}(X) = \sigma^2$ |
|---|---|---|---|---|
| $I_A$ | $[A \subseteq k]$ | | $\mathbb{P}(A)$ | $\mathbb{P}(A)(1 - \mathbb{P}(A))$ |
| $bin(n,p)$ | $\binom{n}{k} p^k (1-p)^{n-k}$ | | $np$ | $np(1-p)$ |
| $geom(p)$ | $(1-p)^{k-1} p$ | | $1/p$ | $(1-p)/p^2$ |
| $Poi(\lambda)$ | $e^{-\lambda} \frac{\lambda^k}{k!}$ | | $\lambda$ | $\lambda$ |

| Förd. | pdf(x) | cdf $= \mathbb{P}(X \leq x)$ | $\mathbb{E}[X] = \mu$ | $\mathrm{Var}(X) = \sigma^2$ | mgf(t) |
|---|---|---|---|---|---|
| $unif[a,b]$ | $1/(b-a)$ | $(x-a)/(b-a)$ | $(b+a)/2$ | $(b+a)^2/12$ | |
| $exp(\lambda)$ | $\lambda e^{-\lambda x}$ | $1 - e^{-\lambda x}$ | $1/\lambda$ | $1/\lambda^2$ | |
| $N(\mu, \sigma^2)$ | $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ | | $\mu$ | $\sigma^2$ | $e^{\mu t + \sigma^2 t^2/2}$ |
| $\Gamma(n, \lambda),\ n \in \mathbb{N}$ | $\frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x}$ | $1 - e^{-\lambda x} \sum_{k=0}^{n-1} \frac{(\lambda x)^k}{k!}$ | $n/\lambda$ | $n/\lambda^2$ | |
| $\chi^2_n = \Gamma(n/2, 1/2)$ | . | . | $n$ | $2n$ | |
| $\beta(a,b)$ | $Cx^{a-1}(1-y)^{b-1}$ | . | $a/(a+b)$ | | |

Glömskeegenskapen: $X \sim exp(\lambda) \implies \mathbb{P}(X > t + x | X > t) = \mathbb{P}(X > x)$

Poissonprocess: Låt $0 < \tau_1 < \tau_2 < \tau_3 < \cdots$ vara en följd av tidpunkter då impulser inträffar. Skriv $T_i = \tau_i - \tau_{i-1}$ för tidsmellanrummen mellan impulser ($\tau_0 = 0$). Om $T_1, T_2, \dots$ oberoende och $exp(\lambda)$-fördelade kallas följden $\{\tau_1, \tau_2, \dots\}$ för en Poissonprocess med intensitet $\lambda$.

$$\tau_n \sim \Gamma(n, \lambda) \qquad X(t) = \text{antal impulser efter tid } t \sim Poi(\lambda t)$$

$$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\,dx \qquad \mathbb{E}[X] = \int_{0}^{\infty} \mathbb{P}(X > x)\,dx \ {\scriptstyle (X \in [0,\infty))} \qquad \mathrm{Var}(X) = \mathbb{E}[(X-\mu)^2]$$

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 \qquad \mathbb{E}[\alpha X + \beta Y + c] = \alpha\,\mathbb{E}[X] + \beta\,\mathbb{E}[Y] + c \qquad \mathrm{Var}(\alpha X + c) = \alpha^2\,\mathrm{Var}(X)$$

$$\mathrm{Cov}(X,Y) = \mathbb{E}[(X - \mu_X)(Y - \mu_Y)] \qquad \mathrm{Var}(X+Y) = \mathrm{Var}(X) + 2\,\mathrm{Cov}(X,Y) + \mathrm{Var}(Y) \qquad \mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y] \ {\scriptstyle (X,Y\ \mathrm{i}}$$

$$\mathrm{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y] \qquad \rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} \qquad r(t) = \frac{f(t)}{G(t)} = \frac{''\,\mathbb{P}(T = t)''}{\mathbb{P}(T > t)}$$

$$M_X(t) = \mathbb{E}[e^{tX}] \qquad M_{X+Y}(t) = \mathbb{E}[e^{tX} e^{tY}] \stackrel{i}{=} M_X(t) M_Y(t) \qquad M_X^{(n)}(0) = \mathbb{E}[X^n]$$

$$f(x,y) = f_{Y|X}(y|x) f_X(x) \qquad \mathbb{E}[Y|X] = \int_{-\infty}^{\infty} y f_{Y|X}(y|x)\,dy \qquad \mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|X]]$$

$$Z \sim N(0,1),\ X = \sigma Z + \mu \implies X \sim N(\mu, \sigma^2) \qquad N(\mu_X, \sigma_Y^2) + N(\mu_Y, \sigma_Y^2) \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$$

$$F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) \qquad \Phi(-x) = 1 - \Phi(x)$$

Chebyshevs o: $\mathbb{P}(|X - \mathbb{E}[X]| \geq \epsilon) \leq \frac{\mathrm{Var}(X)^2}{\epsilon^2}$

CGS: $X_1, X_2, \dots$ iid med $\mu$, $\sigma^2 < \infty \implies \mathbb{P}\left(\frac{\sum_k X_k - n\mu}{\sigma\sqrt{n}} \leq x\right) \to \Phi(x)$

TODO: stora talens lag?

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad s_P^2 = \frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2} \qquad .$$

$$\mu \in \bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad \mu \leq \bar{X} + z_{\alpha}\frac{\sigma}{\sqrt{n}} \qquad \frac{(n-1)s^2}{F^{-1}_{\chi^2_{n-1}}(1-\alpha/2)} \leq \sigma^2 \leq \frac{(n-1)s^2}{F^{-1}_{\chi^2_{n-1}}(\alpha/2)}$$

$$Y \in \bar{X} \pm z_{\alpha/2}\sigma\sqrt{1+\frac{1}{n}} \qquad p \in \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad \mu_X - \mu_Y \in \bar{X} - \bar{Y} \pm z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$$

TODO MLE?
p-värde: sannolikheten att, under $H_0$, erhålla ett så pass extremt resultat
styrka: sannolikheten att förkasta $H_0$ givet $H_A$ sann. $\mathbb{P}_{H_0}(X \notin A) = \alpha$ ger mängd s.a. styrkan är $\mathbb{P}_{H_A}(X \notin A)$.
Linjär regression

$$S_{xx} = \sum(x_k - \bar{x})^2 = \sum x_k^2 - \frac{1}{n}(\sum x_k)^2 \qquad S_{xy} = \sum(x_k - \bar{x})(y_k - \bar{y}) = \sum x_k y_k - \frac{1}{n}(\sum x_k)(\sum y_k)$$

$$Y_k = a + bx_k + \epsilon_k \sim N(a + bx_k, \sigma^2) \qquad \hat{b} = \frac{S_{xy}}{S_{xx}} \sim N(b, \frac{\sigma^2}{S_{xx}}) \qquad \hat{a} = \bar{y} - \hat{b}\bar{x} \sim N(a, \frac{\sigma^2 \sum x_k^2}{nS_{xx}})$$

$$s^2 = \frac{1}{n-2}\sum_{k=1}^{n}(y_k - \hat{a} - \hat{b}x_k)^2 = \frac{1}{n-2}\left(S_{yy} - \frac{S_{xy}^2}{S_{xx}}\right) \qquad b \in \hat{b} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{S_{xx}}}$$

$$Y \in \hat{a} + \hat{b}x \pm F^{-1}_{t_{n-2}}(1-\alpha/2)s\sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{S_{xx}}}$$